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Liquid flow in tubes. II

The transition process under less disturbed inlet flow conditions

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With 19 figures in the text

ABSTRACT

Previous findings (Part I) are confirmed, according to which transition is caused by primary inlet disturbances of finite strength which cause the initiation of turbulent spots before they fade away and according to which the spots during their downstream travel develop into turbulent slugs. It also is confirmed that the fully developed turbulent slug should be regarded as being composed of a turbulent spot (or spots) travelling along the boundaries and of a central core of eddies with decay maintained by a process of continuous diffusion of eddies from the spot into the core (Lindgren 1957, p. 59; 1959, p. 97).

The "splitting" of turbulent slugs as well as the elongation of turbulent streaks, which phenomena have been reported previously (Lindgren 1957, pp. 89–91, 110–111, 121–122, 147–148), are found to be due to the core eddies being brought forward of their generating spots. Some of these always decaying eddies may, before they fade away, initiate the formation of a new spot ahead of the original one. The same process may then repeat itself with the new spots with a probability determined by the flow conditions.

The turbulent streaks and possibly also the self-preserving turbulent flow in general appear to be inhomogeneously maintained by discrete turbulent spots.

Introduction

Recently, qualitative observations were reported on the transition process in stream bi-refringent flow under highly disturbed, entrance-flow conditions (Lindgren 1959). Since, however, the continuous inlet disturbances "drowned" some of the transition features, it was expected that a similar study with a lower disturbance level at the tube inlet would reveal additional details of the transition process and establish a more solid background to further studies.

First, however, it appears necessary to introduce a nomenclature for the phenomena of transition and turbulent flow in order to avoid confusion. It is assumed that a *turbulent spot* can be defined as a wall-near self-preserving turbulent flow-patch from which eddies diffuse into more undisturbed parts of the flow. The name *turbulent flash*, as introduced by Reynolds in 1883, is not applied in the present context. In view of our present knowledge of the phenomenon, discrete, turbulent flow regions appearing in otherwise laminar flows may better be called *turbulent slugs* as

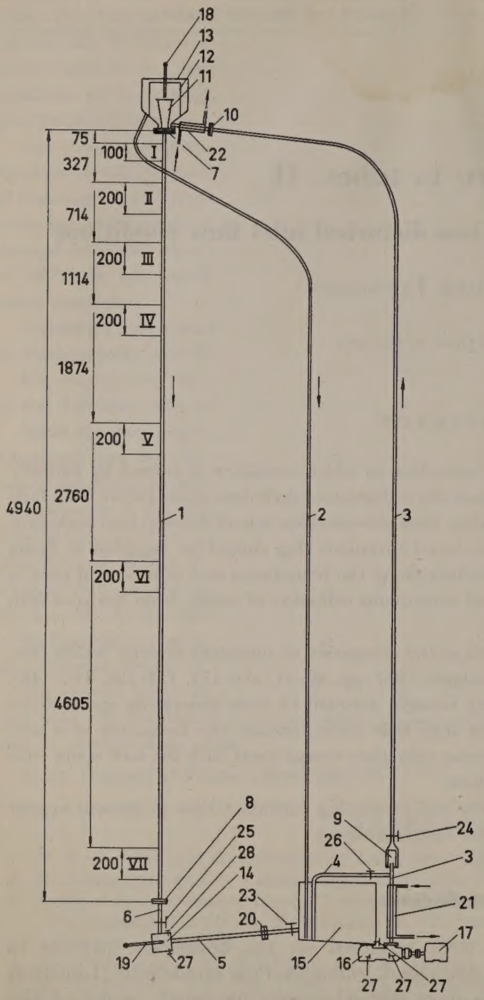


Fig. 1. Arrangement of the experimental flow apparatus

1.	Experiment tube
2.	Overflow return pipe (PVC plastics)
3.	Feeding pipe line (PVC plastics)
4.	Short circuit plexiglass pipe
5.	2 measuring pipes
6.	Joint hose (PVC plastics)
7.	Inlet arrangement from the levelling tank to the experiment tube (see Fig. 2)
8.	Coupling flanches
9.	Glass wool sieve
10.	Glass-fiber-fabric sieve
11.	Outlet trumpet from the levelling tank
12.	Levelling tank
13.	Covering container
14.	Calming tank
15.	Supply container
16.	Circulation pump
17.	Electric motor
18, 19.	Thermometers
20.	2 housings with measuring orifice plates
21, 22.	Heat exchangers
23.	2 regulation cocks
24–26.	Stop cocks
27.	Tap cocks
28.	Bleed cock

Morkovin (1958) does. The turbulent slug may be defined as a turbulent spot (or spots) with an appurtenant turbulent flow-core of decaying eddies, so that at least one part of the entire cross-section of the flow is filled with turbulence. If the turbulent slug extends its length during its travel, it appears suitable to call it also a *turbulent streak*.

In agreement with earlier definitions (Lindgren 1954, 1957, 1959) it appears suitable to call irregular eddy flows, in which the turbulence is not self-preserving, *turbulent*, *disturbed* or *agitated flow*, while the expressions *turbulent channel flow* or *turbulent tube flow* should be reserved for the flow in which the turbulence is is self-preserving.

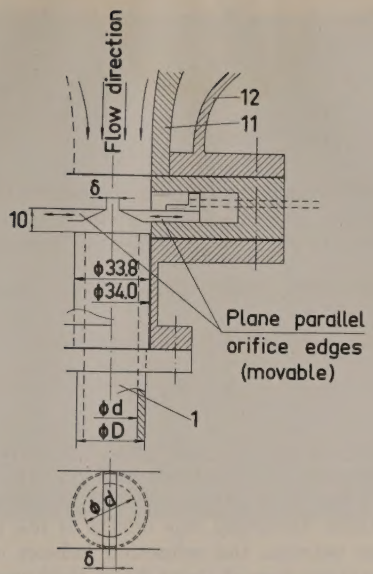


Fig. 2. Arrangement of the experiment tube inlet. 1, experiment tube; 11, outlet trumpet from the levelling tank; 12, levelling tank: d = inner tube diameter = 23.7 mm, D = outer tube diameter = 30 mm. The disturbance edges were removed from the tube opening during the present set of experiments. The inlet disturbances are those caused by a plexiglass rectifier at the trumpet outlet and the plexiglass tube studs placed above the rectifier. (Above the coupling flange the outer diameter of the experiment tube is increased to 33.8 mm in order to avoid a "dead" liquid volume.)

Experimental arrangement

The present experiments were carried out with the same equipment as used previously (Lindgren 1959), including the same experiment tube of 23.7 mm diameter made of plexiglass. The same 2.4 ‰ bentonite suspension was also used. For the sake of the reader's convenience the experimental apparatus is again shown in Fig. 1. The only difference in the experimental arrangement was that the lower part of the trumpet-shaped tank-outlet 11 was equipped with a "honey-comb" rectifier made of short plexiglass tubes and that the rest (upper part) of the trumpet was filled with short studs of plexiglass tubes. This arrangement served the purpose of removing an existing swirl and of reducing the size of the disturbance eddies before entering the tube inlet. The plane, parallel, orifice edges at the trumpet outlet, shown in Fig. 2, were removed from the tube opening so that they caused no constriction of the flow passage. By this arrangement the disturbance level of the entrance flow was considerably reduced, and transition did not occur at Reynolds numbers below a value of about 3200.

As in the previous study, instantaneous flow patterns have been recorded both on still photographs and on motion pictures at the seven observational stations. The distance from the tube entry to the beginning of each observational station was:

For station	I	75 mm,	i.e. ca. 3 tube diameters
	II	327	14
	III	714	30
	IV	1114	47
	V	1874	79
	VI	2760	116
	VII	4605	194

The length of the pictured flow fields is about 100 mm, or 4 tube diameters, at station I, while it is about 200 mm, or 8 diameters, at all the other stations as shown in Fig. 1. Records have been obtained for the following Reynolds numbers:

$$R = 1700, \quad 3290, \quad 3580, \quad 3720, \quad 3850, \quad 4440, \quad 6900,$$

where the Reynolds number is defined by the expression

$$R = Ud/\nu,$$

in which U = mean flow velocity, d = tube diameter, and ν = kinematic viscosity as determined in my paper of 1957 (Lindgren 1957, pp. 18–28).

The experiment tube has been equipped with plexiglass cuvettes filled with water in order to neutralize distorting lens effects of the pictured flow. However, depending on differences between the refraction indexes of water and plexiglass, there is a wall-near layer of the flow of about 2 mm thickness at each side of the flow field from which no visual information whatsoever is received, since no light can pass through there in the view direction.

For a closer description of the experimental arrangement and the recording processes the reader is referred to part I of this series (Lindgren 1959).

Flow development along the tube

It happens that the velocity distribution is rather flat near the tube inlet at a Reynolds number of 1700, as indicated by a broad, black band in the centre of the flow, almost filling up the whole cross-section except rather near the walls where light parts indicate the presence of shear or of velocity gradients. In Fig. 3 this state of affairs is illustrated by picture *a* showing the flow pattern at Stn I. We also note that the broad, black centre-stripe decreases in width downstream, so that the velocity profile visually appears fully developed at the lower part of Stn II as shown by picture *b*, reaching its final stage of development upstream of Stn III in picture *c*.

Fig. 4 illustrates the appearance of flow patterns at a Reynolds number of $R \approx 3290$. Here we can see the presence of primary eddy disturbances at Stn I (picture *a*) next to the tube inlet. We note that these entry disturbances are rather well collected within a central flow core which diminishes its cross-section with increasing distance from the tube inlet. (See pictures *a*, *b*, *c* and *d* of Stns I, II, III and IV.) Picture *d* (according to which only weak disturbance fragments remain at Stn IV) furthermore shows the same asymmetric pattern as is also to be noted at Stn IV for the less disturbed flow at the lower Reynolds number $R \approx 1700$ (Fig. 3*d*). No primary disturbances were ever observed to reach as far as Stn V (picture 4*e*). Still there remains a steady asymmetry in picture *e* at Stn V which is a little stronger than

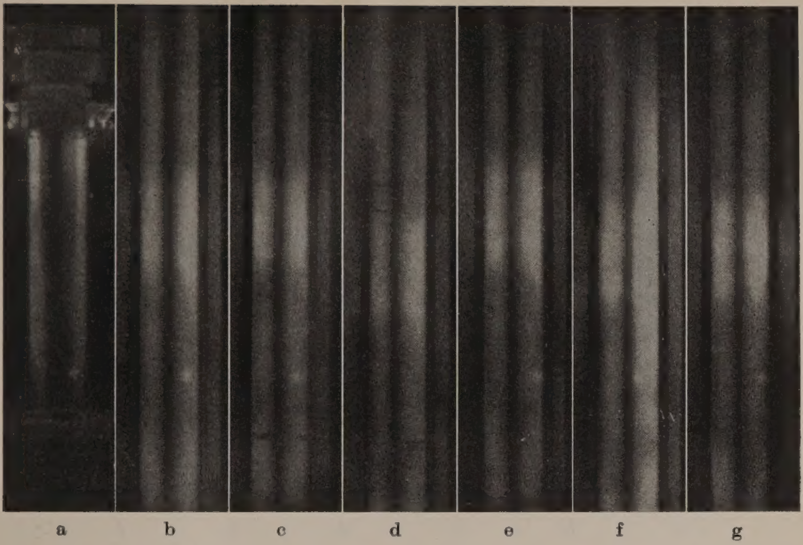


Fig. 3. The appearance of the stream bi-refringent flow at the seven observational stations from left to right for a Reynolds number of $R \approx 1700$. Note the wide dark stripe within the flow core at Stn I (picture *a*) indicating a flattened velocity profile. At Stn II (picture *b*) the black stripe is seen to reach its equilibrium width in the middle of the picture. At Stn III (picture *c*) we note the first tendencies for the permanent black line to reach an asymmetric position within the tube, an asymmetry which has increased at Stn IV (picture *d*). However, at Stn V (picture *e*) the asymmetry has decreased and at Stn VI (picture *f*) full symmetry is again established, to remain further downstream at Stn VII (picture *g*). This displacement of the black "centre" line, indicating zero shear stress there, evidently represents a real flow asymmetry, as can be concluded from recent findings by Reshotko (1958) in Pasadena.

The flow field is in reality bounded by the light lines seen inside the tube walls (the outer dark stripes). The inner dark stripes are due to total reflection of light within wall-near layers, caused by differences between the refraction indexes of the water and the tube walls. Elementary calculations indicate that in the present experiments no perpendicular light would pass through a wall-near layer of about 2 mm thickness. Therefore, no visual information whatsoever regarding the flow texture in profile is received from within these wall-near layers.

The location and length of each pictured tube section, in this and the following figures, are given in Fig. 1.

the asymmetry shown at Stn V by Fig. 3*e* for the lower Reynolds number. Again at Stn VI (picture *f*) full symmetry is established and remains farther downstream. At this Reynolds number only on rare occasions were turbulent spots triggered off by the inlet disturbances with a subsequent development into turbulent streaks.

In view of recent findings by Reshotko (1958) it appears that the optically noted asymmetry probably corresponds to a real flow asymmetry, due to slight lateral deflections of the pipe. However, this asymmetry as yet has not proved to have any noticeable effect on the transition process.

When increasing the Reynolds number to $R \approx 3580$, turbulent spots appeared now and then in the flow and developed into turbulent streaks. The spots mostly seemed to be triggered off in the regions of Stns II or III and to be in different stages

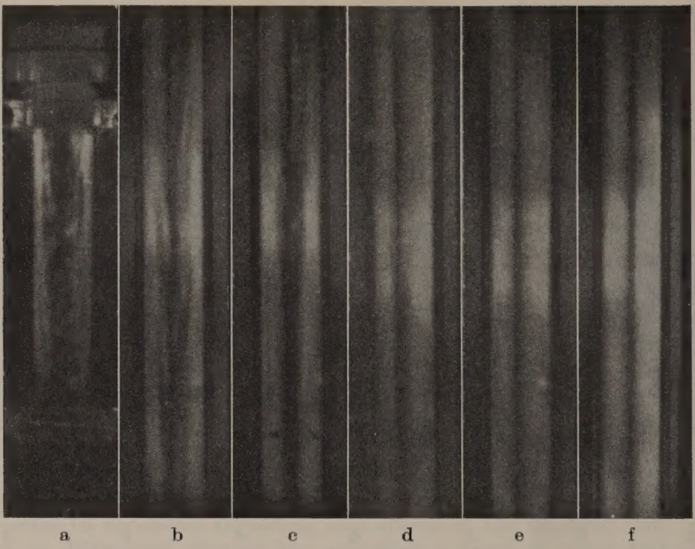


Fig. 4. Here the Reynolds number of the flow is about $R \approx 3300$ and there are disturbance eddies present, of which some large ones remain as far downstream as Stn IV (picture *d*) before they fade away. On some rare occasions there was observed the formation of turbulent spots at Stns III or IV in this flow, which spots developed into turbulent streaks before reaching Stn V (picture *e*). From there the flow otherwise appears purely laminar.

of development around Stns III and IV, while they had always grown into turbulent streaks before reaching Stn V. In agreement with previous findings (Lindgren 1957, pp. 90–94, 147–148), the streaks extended themselves during their downstream transport, so that they were longer when passing Stn VI than Stn V and had, of course, further increased their lengths at Stn VII. Note, however, that the length of the various turbulent streaks might vary considerably for the same flow at the same location. Fig. 5 serves as an illustration of these circumstances. Also here the inlet disturbances at Stns I and II stay within a central flow-core as already noted in Fig. 4 for flow at the lower Reynolds number $R \approx 3290$. However, the disturbance core is perhaps a little thicker at the higher Reynolds number. In picture *c* (Stn III) we note the primary stages of development of a turbulent spot along the left-hand wall on the second half of the pictured tube section. The characteristics of this spot suggest that it travels along that part of the tube wall which faces the viewer, while picture *d* (Stn IV) shows a later stage of development of a turbulent spot travelling along the right-hand half of the tube wall situated opposite the viewer. Pictures *e*, *f* and *g* show in this order the front, the rear and the front of turbulent streaks passing corresponding Stns V, VI and VII.

Fig. 6 shows a cinematographic record of the first phases of development of a turbulent spot at Stn III in the same flow as above ($R \approx 3580$). We note how the finger-like spot first appears at the upper part of the second picture of the series and how it then travels along the far side of the tube wall a little towards the left.

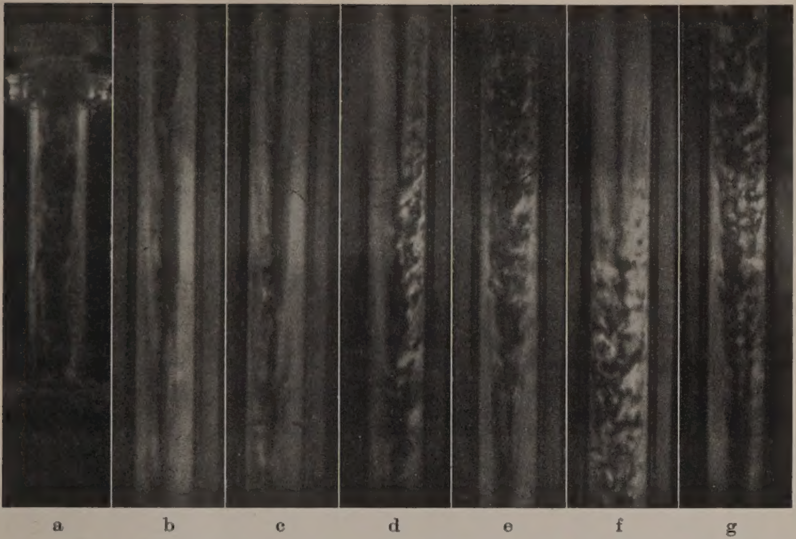


Fig. 5. When the Reynolds number was increased to about $R \approx 3580$, turbulent spots were triggered off now and then at Stations III and IV (pictures c and d). At Stn I (picture a) only primary disturbances with decay appeared, while knotting-up tendencies in the disturbance eddies were observed at Stn II (picture b), indicating the first signs of triggering off turbulent spots. Picture c actually shows the first stages of development of a turbulent spot at Stn III. The flow pattern suggests that this initial spot is travelling along the left part of the tube wall which is situated nearest to the viewer, while the more developed turbulent spot at Stn IV shown in picture d seems to travel along a right-hand part of the tube wall located on the other side of the tube centre relative to the viewer.

The turbulent spots always developed into continuous turbulent streaks before reaching Stn V, as can be seen in picture e, which shows the front of a continuous turbulent streak at this station, while picture f shows the rear of a similar turbulent streak passing Stn VI. Picture g shows a similar pointed front of a turbulent streak at Stn VII appearing also at Stn V.

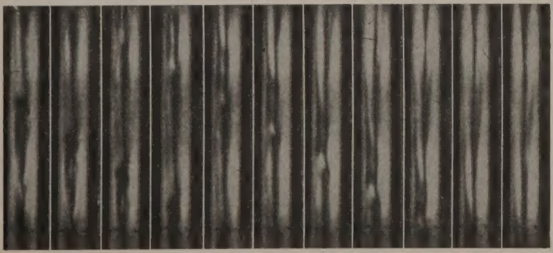


Fig. 6. This figure demonstrates the first phases of development of a turbulent spot appearing at Stn III for flow of the Reynolds number $R \approx 3580$. The formation and downstream transport of the finger-like spot region can be followed from the second until the tenth picture of the sequence with a time interval of $\frac{1}{12}$ of a second between each picture. The entire series of 11 pictures thus represents a time interval of $\frac{5}{6}$ second. The exposure time for each picture is about 2×10^{-6} second.

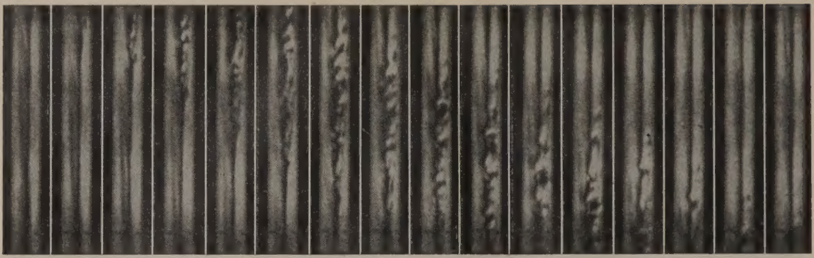


Fig. 7. In this figure we can follow the passage of a turbulent spot in later stages of development than in Fig. 6. This record also is obtained at Stn III for a Reynolds number of $R \approx 3580$. The spot travels along walls located on the other side of the tube centre relative to the viewer. There is $\frac{1}{12}$ second between each exposure. The complete sequence of 16 pictures from left to right thus represents a time interval of $1\frac{1}{4}$ second.

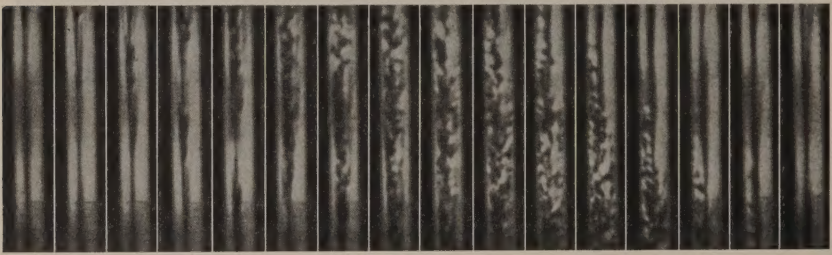


Fig. 8. Here we note the passage of a turbulent slug which is just developing out of a turbulent spot at Stn IV for the same Reynolds number, $R \approx 3580$, as in Figs. 6 and 7. Note the asymmetric form of the slug body. $\frac{1}{6}$ second interval between each exposure. The entire sequence of 16 pictures represents a time interval of $2\frac{2}{3}$ seconds.

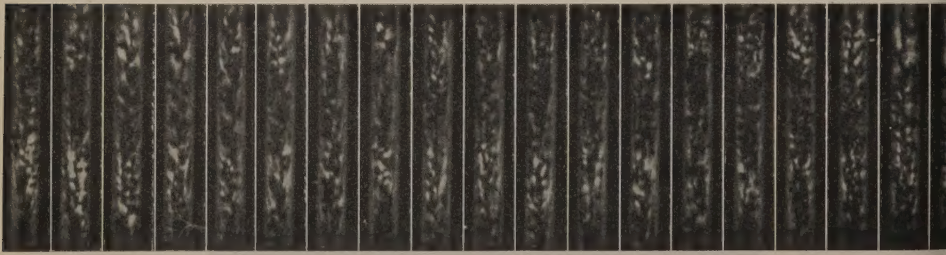


Fig. 9. The figure shows various phases and parts of a turbulent streak as it passes Stn V at the same Reynolds number, $R \approx 3580$, as in the previous three figures. The first two pictures on the left show some parts of the pointed front of the streak, while in the last two pictures on the right its rear has come in view. Studying the various pictures and comparing them with each other reveals how discrete alternating wall-near turbulent spots travel down-stream, along varying parts of the tube-wall circumference, interfering with each other so that an inhomogeneous and mixed turbulent flow-pattern appears. There is a time delay of $\frac{1}{6}$ second between each exposure and the complete sequence of 19 pictures thus represents an interval of 3 seconds.

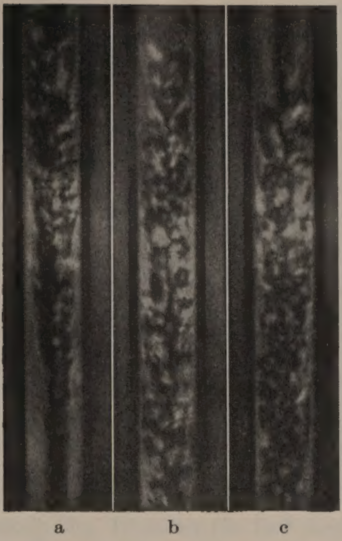


Fig. 10. This figure gives more details regarding the inhomogeneous eddy distribution within a turbulent streak than can be seen in Fig. 9. It shows the asymmetrically pointed front, the middle part, and the rear of a streak when passing Stn VII within the same flow as is pictured in Fig. 9.

Fig. 7 in turn shows the passage of a spot at Stn III during a later stage of development than in Fig. 6 for flow at the same Reynolds number. This spot evidently travels along the right-hand side of the tube wall-surface which is located opposite the viewer.

Fig. 8 shows the passage of a turbulent slug followed by a turbulent spot at its rear when passing Stn IV for the same flow as before ($R \approx 3580$). From the diffuse turbulence structure of the slug we can infer that the eddies of the spot are moving along parts of the tube wall which face the viewer, while the rear spot appears to be travelling along the left-hand wall located on the far side of the tube.

Fig. 9, finally, shows the passage of a turbulent streak at Stn V for flow at the same Reynolds number, $R \approx 3580$, as in Figs. 5–8. A close examination of the pictures in this figure reveals that the eddy distribution within the streak is not homogeneous. Instead, support is lent to earlier findings that the apparently continuous, turbulent streak is maintained by an increasing number of separate turbulent spots which preserve themselves during their travel and from which turbulence continuously diffuses into more central parts of the flow. Fig. 10 gives more details of the flow structure. It shows the front, a sample of the middle part and the rear of typical turbulent streaks at Stn VII for the same Reynolds number as in Fig. 9.

In Fig. 11 the Reynolds number of the flow has been increased to $R \approx 4440$. We note that the disturbed inlet flow at Stn I (picture *a*) still shows about the same characteristics as for flows at lower Reynolds numbers, while at Stn II (picture *b*) the primary disturbances are evidently mixed with more "textural" eddies along the wall opposite the viewer indicating the first signs of the birth of turbulent spots which are frequently developed at Stn III, one spot of which is shown in picture *c*. These

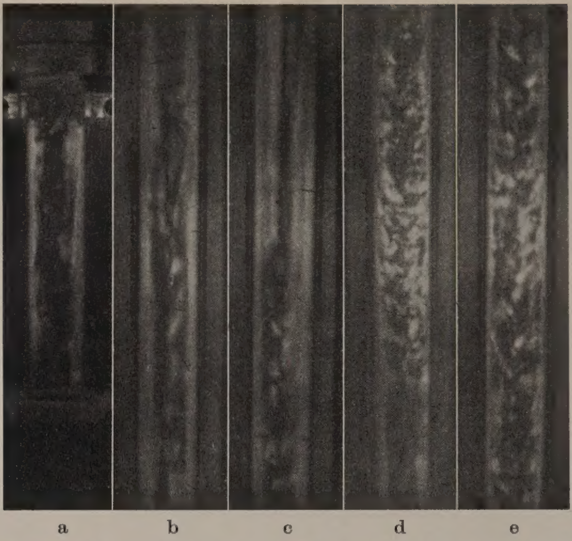


Fig. 11. When the Reynolds number was increased to $R \approx 4440$, the breakdown of the primary flow disturbances and the formation of turbulent spots took place at Stns II and III, while still only primary disturbances were present at Stn I. Thus, well-developed long turbulent streaks appeared at Stn IV, while the flow was continuously turbulent already at Stn V (and further downstream, of course).

In picture *a* we see the primary disturbance pattern at Stn I, while picture *b* shows a textural flow pattern which is ripe for breakdown and spot formation at Stn II. In picture *c* (Stn III) we note the formation and passage of a turbulent spot which seems to travel along the left part of the tube wall located on the near side of the tube. Picture *d* shows the first portion of a turbulent streak passing Stn IV, while picture *e* shows parts of the continuous turbulent flow pattern at Stn V.

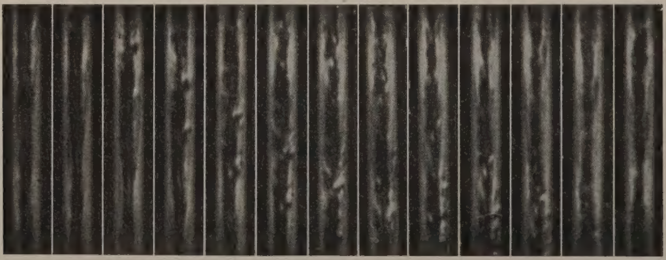


Fig. 12. This figure shows the first stages of development of a turbulent spot at Stn II in a flow of $R \approx 4400$. Note the two tiny "balls" on the right in the upper part of the third picture which must have been formed during the $\frac{1}{12}$ second interval between the exposure of the second and the third pictures. That this is so will be realised if we follow the downstream transport of the "balls" in the following pictures, which indicate that they must otherwise have been present also in the upper part of the second picture. Further it should be noted how these balls begin to deterioriate in the sixth and seventh pictures, developing into a wall-close elongated spot which is further retarded and first departs from the pictured flow field in the eleventh picture. Time interval between each exposure is $\frac{1}{12}$ second and the complete sequence represents a time interval of 1 second.

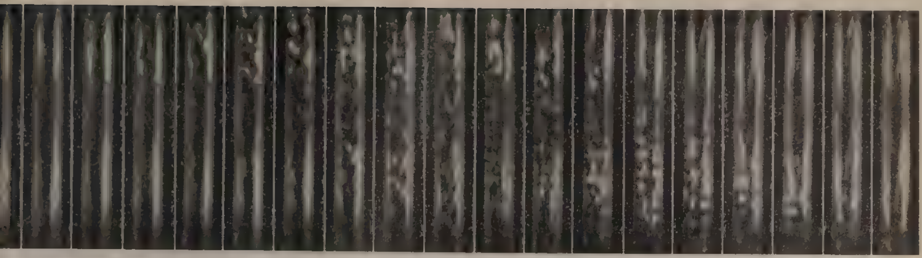


Fig. 13. This figure shows the passage of a turbulent spot in the later stages of development at Stn III for the same flow as shown in Fig. 12 ($R \approx 4440$). Note, however, that eddies diffusing from the spot begin to fill out the entire flow cross-section in the twelfth and thirteenth pictures, and that the rear of a slug has completely developed before it leaves the recorded flow field in the last 4 pictures.

spots develop into turbulent streaks before reaching Stn IV as shown in picture *d*, where we note the front of a turbulent streak. Already at Stn V the flow has developed into continuous turbulence, a pattern of which is shown in picture *e*. Here, too, we note the inhomogeneity of the turbulence texture.

The sequence of pictures in Fig. 12 shows the first stages of development into a turbulent spot at Stn II for the same Reynolds number as above, $R \approx 4440$. In the first two pictures on the left we note only a general inlet disturbance, but in the third picture we observe how two "balls" have been formed at the upper part of the pictured tube section slightly to the right of the tube centre along the wall opposite the viewer. We can then follow the passage of these "balls", as they proceed downstream until the sixth picture, after which they begin to disappear, forming a turbulent spot. This spot travels more slowly along the pictured tube section on the lower right, and has disappeared completely from the flow field in the last picture of the sequence. Fig. 13 shows how a turbulent spot passes Stn III for flow at the same Reynolds number, $R \approx 4440$, and develops into a turbulent slug before the rear of it passes out of sight. It is interesting to note that the spot acquires rather quickly the characteristics of a turbulent slug with a somewhat square rear and a pointed front.

When the Reynolds number was increased to $R \approx 6900$ there was a frequent flash-like formation of turbulent spots appearing within the disturbed inlet flow at Stn II, while the disturbed flow pattern at Stn I still looked about the same as for the lower Reynolds numbers. Fig. 14 shows this state of affairs. In picture *b* we note turbulent spots on the upper left and lower right of the pictured tube section. There is also to be noted the very end of another spot far down to the left in this picture. The turbulence is fully developed already at Stn III and its structure is shown in picture *c*. Fig. 15 shows a sequence of pictures of the same flow at Stn II, in which can be observed the formation and downstream transport of turbulent spots quickly developing into overlapping slugs. Under the present circumstances the frequency of the flash-like formation of spots at Stn II was of the order of 2–4 times per second.

Fig. 16, including also three close-up photographs, serves the purpose of giving a more detailed description of the appearance of turbulent spots at Stn II for the Reynolds number $R \approx 6900$. In the middle, a little to the left in picture *e*, we note the characteristic knotting-up of the flow-texture which implies the flow breakdown

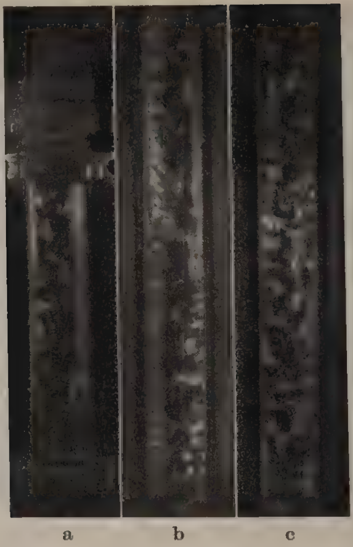

a b c

Fig. 14. When the Reynolds number was increased to about $R \approx 6900$, the disturbance level at the tube inlet was rather high, including also small-scale eddies, as shown in picture *a* of Stn I. At Stn II was noted the flash-like formation of turbulent spots along the walls within the disturbed flow region with a frequency of about 2–4 times per second. Picture *b* shows such spot formations. One spot can be seen on the upper left while another is present on the lower right; the latter seems to interfere with the extreme rear of a turbulent spot which can be seen farthest down to the left in the picture. The turbulence continuity was fully developed already at Stn III, a pattern of which is shown in picture *c*.

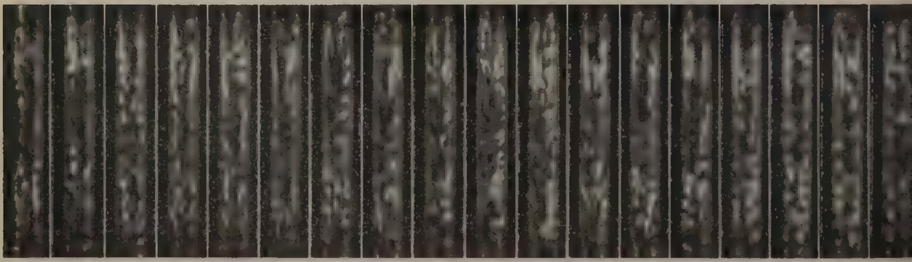


Fig. 15. This figure shows a cinematographic record of the flash-like formation of turbulent spots at Stn II for flow of the Reynolds number $R \approx 6900$. The first five pictures show a spot passage along the left-hand side of the tube, while the next five pictures show a similar spot pattern along the right-hand side. In the next two pictures there is more developed turbulent flow, while the following two pictures again show spot formation in the upper right part of the tube. The last four pictures show more or less continuous turbulence, which, however, appears by no means homogeneous. Time interval between each exposure $\frac{1}{12}$ second.

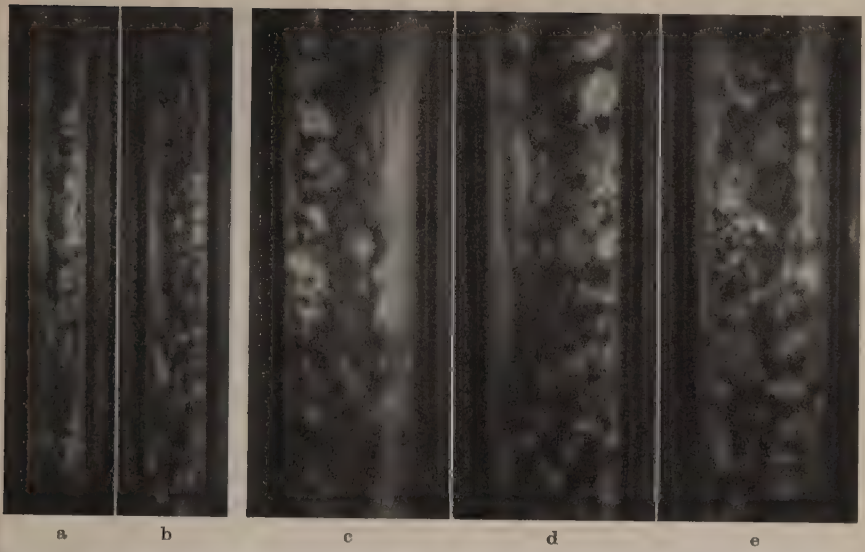
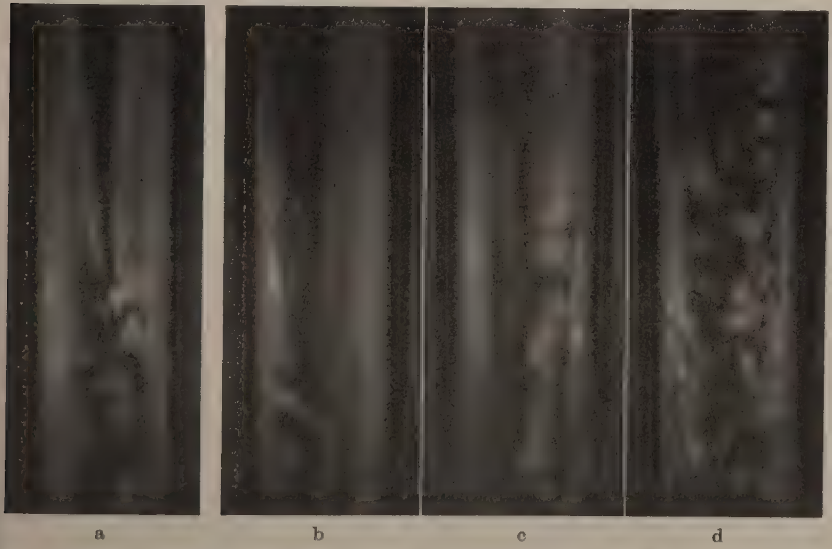


Fig. 16. This figure further illustrates the process of spot formation in a flow of $R \approx 6900$ at Stn II. The close-up photographs are specially instructive. In the middle of picture *e*, a little to the left, we further note a large-grain flow texture, indicating the burst of another spot beside the already existing one.



ig. 17. Picture *a* shows the break-down pattern of a primary disturbance, preceding the birth f a turbulent spot at Stn III for a Reynolds number of about $R \approx 3580$. Pictures *b*, *c* and *d* oncern flow features at the same station for a Reynolds number of $R \approx 3850$. Picture *b* shows breakdown pattern, as does picture *a*, but more in profile, while picture *c* shows the second half f a turbulent spot in an early stage of development. Picture *d*, finally, shows the later stages of development of a turbulent spot before it can be identified as a turbulent slug.

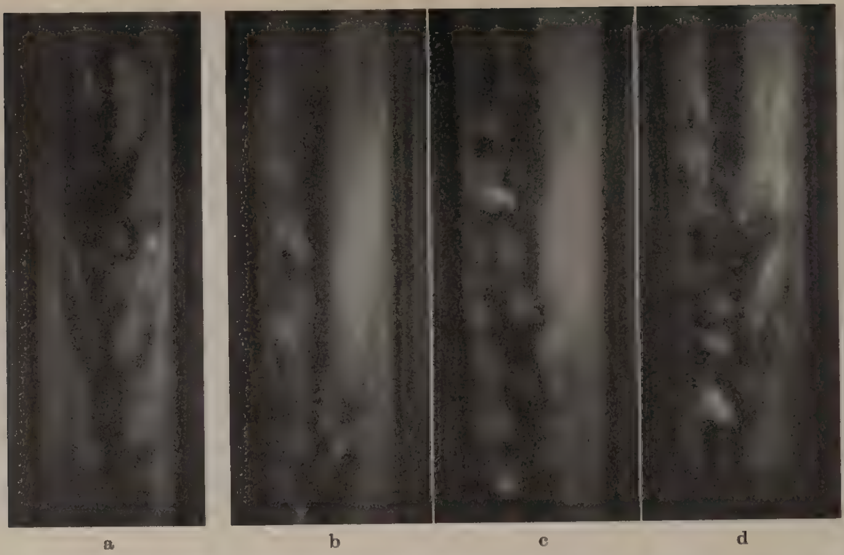


Fig. 18. Picture a shows a turbulent spot in later stages of development towards a turbulent slug when passing Stn IV in a flow of the Reynolds number $R \approx 3580$, while pictures *b*, *c* and *d* show various stages of development of turbulent spots at the same station, but for a Reynolds number of about $R \approx 3720$. The last picture is a good demonstration of a spot which has expanded sideways so that it also travels along parts of the tube wall situated near the viewer, while the main spot body is still seen more or less in profile.

which may initiate the subsequent formation of a spot. In this case the breakdown takes place along the tube wall facing the viewer, while an already well-developed turbulent spot is passing along the right-hand wall opposite the viewer in the same picture.

The characteristic knotting of the flow pattern shown in Fig. 16 *e* above, which indicates the initiation of a turbulent spot on the tube wall facing the viewer, is visible also in Fig. 17, picture *a*, where it occurs at Stn III for a Reynolds number of 3580, while picture *b* of the same figure also shows the same phenomenon in a sideways view at Stn III, but for a Reynolds number of $R \approx 3850$. Picture *c* of the same figure shows the latter half of a spot in an early stage of development along the right-hand wall opposite the viewer, while picture *d* shows the central parts of a more developed spot in contact with the tube wall facing the viewer. Both pictures *c* and *d* were obtained with the same flow of $R \approx 3850$ at Stn III.

Fig. 18 is an illustration of the appearance of turbulent spots at Stn IV. Picture *a* shows a spot appearing in a flow at $R \approx 3580$ along parts of the tube wall facing the viewer on the right. Pictures *b*, *c* and *d* show the appearance of turbulent spots within flows at the Reynolds number $R \approx 3720$. The spots in pictures *b* and *c*, which are in earlier stages of development, travel along the left-hand tube wall opposite to the viewer, while the spot shown in picture *d*, which seems to be in contact also with the tube wall facing the viewer, is in a later stage of development and actually maintains a turbulent slug of the asymmetric type.

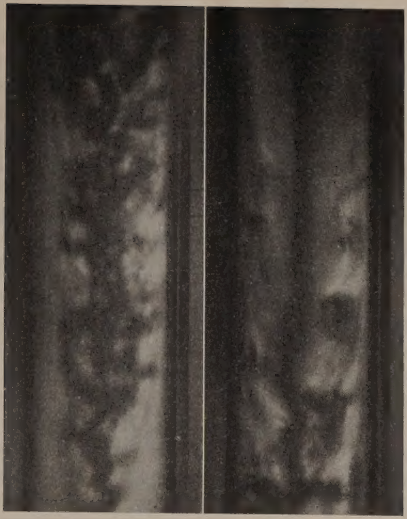


Fig. 19. This figure gives a clear view of the eddy structure at the front and the rear of a turbulent streak passing Stn VI in a flow of the Reynolds number $R \approx 3720$.

Fig. 19 finally shows the front and the rear of a turbulent streak passing Stn VI at a Reynolds number of $R \approx 3720$. We note that the asymmetrically pointed front of picture *a* is maintained by a turbulent spot travelling along the right-hand side of the tube, while the somewhat hollow rear in picture *b* appears to be maintained more symmetrically by two spots more or less facing each other.

Conclusions

From the reported observations and the illustrations reproduced we may conclude that *transition in tube flow implies the formation of turbulent spots initiated by primary disturbances of finite magnitude.* This state of affairs is in full agreement with observations reported by Kuethe 1956 and Leite 1959 and is also valid in the case of Tollmien wave instabilities, as can be inferred already from Schubauer & Skramstad's experiments from 1943, but still better from Klebanoff's recent boundary-ayer investigations.[1] The turbulent spots may gradually develop into turbulent slugs which further on might fade away, split or grow into turbulent streaks depending on flow and channel conditions.

Generally speaking, *the turbulent slug seems to be maintained by one or several, wall-near, self-preserving turbulent spots* which travel with an average velocity determined by the Reynolds number and other flow conditions. From the spots eddies continuously diffuse into more central parts of the flow. Since the flow velocity is higher in the flow centre than close to the walls the always decaying core eddies reach certain distances ahead of their generating bodies before they fade away. They reach farthest ahead in the middle of the flow where the velocity is highest. This accounts for the fact that the slugs always have a pointed front and a somewhat hollow rear.

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[1] Personal communication in spring 1958.

The decaying core eddies emanating from a turbulent spot may, before they fade away, initiate the formation of a new turbulent spot ahead of the original one and so forth. Within the lower transition region this process may result in an occasional formation of a new spot which might develop into a turbulent slug ahead of the original one and then the process perhaps may be repeated for the new slug and so on. In this way one turbulent slug causes the formation of one or several slugs ahead of itself during its travel with the flow. We thus have arrived at a reasonable explanation of the mechanism of the "splitting" of turbulent slugs, a phenomenon which has been described previously (Lindgren 1957, pp. 89, 110–111, 147–148).

The probability of the formation of new spots according to the process described above, must necessarily increase both in space and time with increasing Reynolds numbers. This scheme implies complex mixing processes and results in a more or less "continuous" elongation of turbulent streaks travelling downstream with the flow (Lindgren 1957, pp. 89, 90–92, 128, 148). The elongation of the streaks must evidently appear in somewhat discrete jumps. However, as the Reynolds number increases, the elongation will occur more and more continuously.

From the description above we may infer that *the "splitting" of turbulent slugs at lower Reynolds numbers, R, and the "elongation" of turbulent streaks at higher R actually represent various stages of a random function of one and the same process of spot formation.*

From the present investigation it appears that *the turbulence structure in an expanding turbulent streak should not be considered to be homogeneous either in space or in time but to be composed of wall-near turbulent spots which continuously regenerate themselves and maintain a core of dissipating eddies which initiate the formation of new spots ahead of the momentarily existing turbulence front.*

Provided the Reynolds number is well above its "critical" value, R_k, and there is a high disturbance level at the tube inlet, continuous tube turbulence will develop and remain after some entrance length, while in case of low disturbance levels turbulent streaks alternating with laminar intervals will appear in the flow within certain lengths of the tube. It is then conceivable that there should be no qualitative difference between the basic structure of continuous turbulent flow and extending turbulent streaks at the same Reynolds number within the same channel and at the same downstream location. If this should be true, *continuous turbulent tube flow also should be inhomogeneously maintained by a number of self-preserving discrete turbulent spots,* though probably in a larger number than in the corresponding case of turbulent streaks, since, depending on the necessarily higher disturbance level of the inlet flow in the case of continuous turbulent flow, there must be a correspondingly higher frequency of initial spot generation than in the case of turbulent streaks.

The question now arises, whether core eddies from turbulent spots located behind would cause the formation of new spots in the interspaces between spots located more downstream, so that one huge homogeneous turbulent wall-spot would develop sooner or later with increasing travelling distance? At present this question is open for discussion and further studies.

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